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Year 12 sample questions for entry to A Level Mathematics

For entry onto A-Level Mathematics (or Further Mathematics) we expect prospective students to demonstrate that they can use the mathematical skills that are required to answer the questions in this document. Note that these exact questions will not appear in the entry test.

surds

Q1. Rationalise.

(a) $\frac{1}{\sqrt{7}}$

(b) $\frac{2}{5\sqrt{3}}$

(c) $\frac{1}{\sqrt{x}}$

(d) $\frac{1}{3-\sqrt{7}}$

(e) $\frac{2\sqrt{5}}{\sqrt{3}-\sqrt{2}}$

(f) $\frac{1}{\sqrt{p}-\sqrt{q}}$

Q2. Simplify the following expressions. You must show all of your working.

(a) $\sqrt{2} \times \sqrt{18}$

(b) $\sqrt{45} - 3\sqrt{20} - 3\sqrt{5}$

(c) $\sqrt{50} + 2\sqrt{8} - \sqrt{18}$

(d) $2\sqrt{108} - 2\sqrt{48} - 3\sqrt{75}$

Q3. Expand the following brackets. You must show all of your working.

(a) $(2 + \sqrt{5})^2$

(b) $(3 - 4\sqrt{2})(7 + 3\sqrt{2})$

Q4. Rationalise

$$\frac{1 - \sqrt{2}}{(3 + 2\sqrt{2})^2}$$

indices

Q5. Express the following algebraic expressions in terms of a.

(a) $\sqrt{a^{-11} \times a^3}$

(b) $\left(\frac{1}{3}a^{-2}\right)^4 \div \left(\frac{1}{9}a^{-3}\right)^3$

Q6. Solve

$$\sqrt{3} \times 9^{3x-1} = \sqrt{27}$$

Q7. Simplify

$$\left(\frac{81x^{\frac{4}{3}}}{16p^{\frac{8}{5}}}\right)^{-\frac{3}{4}}$$

Q8. Evaluate

$$\left(\sqrt[9]{5^3}\right)^{-3}$$

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percentages

Q9. What percentage of the number 62.5 is the number $11\frac{1}{4}$.

Q10. 24% of a number is 16. Find the number.

Q11. Umberto has a large number of books on his shelf. One day he concluded he got too many of them and got rid of 25% of the books. He kept on buying new books so soon the number increased by 14%. Then he had 684 books. How many books did Umberto have initially? Which number of books - initial or final - was smaller? By how many percent?

probability

Q12. A box contains 4 red balls and 8 white balls. A ball is randomly drawn out of the box, its colour is noted and the ball is then replaced. Then another ball is drawn.

(1) Find the probability that both balls are white.

(2) Find the probability that the two balls are of different colours.

Q13. The group of students consists of 25 women and 30 men. Sixteen people in the group are red-haired. Sixteen women in the group do not have red hair. Draw the Venn diagram to show the situation and hence write down the number red-haired men.

rearranging formulae

Q14. Make y the subject

(a) $\frac{3-y}{a+y} = \frac{1}{b}$

(b) $\sqrt{\frac{a^2+y}{b^2+y}} = \frac{1}{c}$

quadratics and equations

Q15. Factorise

(a) $x^2 + 4x$

(b) $x^2 - 9$

(c) $4x^2 - 49$

(d) $2x^2 - 50$

(e) $x^2 - 2x - 3$

(f) $6x^2 - 5x + 1$

Q16. (a) Solve the following equation using the quadratic formula

$$x^2 - 3x + 1 = 0$$

Give your answers to three decimal places.

(b) Solve the following equation using the quadratic formula

$$2x^2 + 9x - 1 = 0$$

Give exact answers

(c) Solve the following equation by factorisation

$$2x^2 + 5x + 2 = 0$$

Q17. Complete the square

(a) $x^2 - 4x + 1$

(b) $x^2 - 3x + 10$

(c) $-x^2 + 5x - 3$

(d) $3x^2 + 12x - 1$

(e) $6x^2 + x - 5$

Q18. Solve the following equation by completing the square

$$-4x^2 - 8x + 1 = 0$$

Q19. *Quadratic graphs and their equations*

(a) Find the equation of a parabola that has x-intercepts $(1, 0)$ and $(5, 0)$ and passes through point $(2, 6)$. Give answer in the form $y = ax^2 + bx + c$.

(b) Find the equation of a parabola that has a vertex at $(4, -2)$ and passes through point $(2, -6)$. Give answer in the form $y = ax^2 + bx + c$.

Q20. *Quadratic inequalities*
Solve

(a) $x^2 < x + 12$

(b) $x^2 - 5x + 6 > 0$

(c) $2x^2 + 3x - 2 < 0$

Q21. *Solving equations involving algebraic fractions* Solve

(a) $\frac{1}{x-1} - \frac{2}{x-2} + \frac{3}{2} = 0$

(b) $\frac{3}{x-2} = \frac{x-2}{27}$

(c) $\frac{x^2}{x+1} = 2$

Q22. *Simplifying and combining algebraic fractions*
Simplify

(a) $\frac{1-4x^2}{9x^2-6x+1} \div \frac{4x^2+4x+1}{1-9x^2}$

(b) $\frac{6x^2-16x+8}{3x+2} \times \frac{12x^2+5x-2}{5x-10}$

Q23. Express as a single fraction in its lowest terms.

$$\frac{2x - 5}{25x^2 + 20x + 4} - \frac{5x}{15x^2 + 6x}$$

Q24. *Simultaneous equations* Solve

$$x^2 + 2xy = 20$$
$$y - 2x = 1$$

Give your answers to two decimal places.

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Factorising/solving cubic equations

Q25. Factorise

(a) $x^3 + 2x$

(b) $2x^3 - 3x^2 - 2x$

Q26. Solve

(a) $8x^3 - 1 = 0$

(b) $x^3 + 5x^2 + 6x = 0$

straight lines

Q27. Find the mid-point of the line segment AB for $A(9, 2)$ and $B(3, 12)$.

Q28. Find the distance between $A(-4, 5)$ and $B(-1, -4)$.

Q29. Find the gradient (slope) of the line segment AB. Give the answer as an integer or an exact, simplified fraction.

(a) $A(2, 5)$, $B(5, 9)$

(b) $A(-2, -2)$, $B(-6, 4)$

Q30. Find the missing coordinate of point $A(6, y)$ knowing that it lies on the line $y = \frac{2}{3}x + 3$.

Q31. Find the line parallel to $k : y = \frac{1}{4}x + 2$ and passing through $A(4, \frac{1}{2})$.

Q32. Find the line perpendicular to $k : y = \frac{7}{3}x + 3\frac{2}{3}$ and passing through $A(-2, -1)$

Q33. Find the general equation of the line **parallel** to $k : x-2y+3 = 0$ and passing through point $P(-3, -2)$

Q34. Find the general equation of the line **perpendicular** to $k : 3x - y - 2 = 0$ and passing through point $P(6, 4)$

Q35. Find the equation of the line passing through $A(6, 5)$ and $B(-3, -1)$.

Q36. Find the equation of the line that is perpendicular to the line passing through $A(1, 4)$ and $B(4, 10)$, and which passes through $(-1, 10)$.

Q37. Find the equation of a perpendicular bisector of the line segment AB, where $A = (-8, 5)$ and $B = (-2, -1)$.

Q38. Solve the simultaneous equations.

(1) $\begin{cases} x - 3y + 5 = 0 \\ 2x + y + 3 = 0 \end{cases}$ (2) $\begin{cases} 2x + 3y + 5 = 0 \\ 3x - 4y = 52 \end{cases}$

Q39. *Applications of linear equations*
Let A and B be points with y-coordinate 6 on lines $y = -\frac{1}{3}x + 4$ and $y = 2x - 3$ respectively.

(i) Find the x-coordinates of A and B.

(ii) Find the coordinates of point C - the intersection of the lines.

(iii) Find the area of triangle ABC.

Functions

Q40. Find the largest possible domain and the range of the following functions.

(a) $f(x) = -4x + 3$

(b) $f(x) = \sqrt{x-3}$

(c) $f(x) = x^2 - 6x - 9$

Q41. Find $f(a)$ for given function f and the value of a.

(1) $f(x) = 2x + 5, \qquad a = -2$

(2) $f(x) = \sqrt{3-x}, \qquad a = -6$

(3) $f(x) = 3 + \frac{2}{x-1}, \qquad a = 5$

Q42. The functions $f(x)$ and $g(x)$ are defined as

$$f(x) = x^2 + 3, \qquad g(x) = x + 2$$

(a) Find $fg(4)$

(b) Find $gf(4)$

(c) Find $fg(x)$

(d) Solve

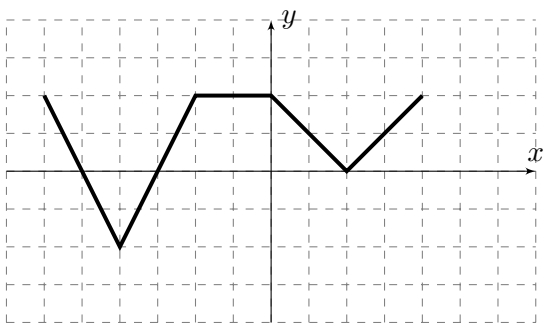
$$gf(x) = 21$$

Q43. The function $f(x)$ is defined as

$$f(x) = \frac{x-2}{x+1}$$

Find $f^{-1}(x)$.

Q44. The diagram below shows the graph of $y = f(x)$.



Graph the functions with equations shown.

(1) $y = f(-x) - 2$

(2) $y = -f(x+1) - 1$

(3) $y = f(\frac{x}{2})$

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trigonometry

Q45. Find the other two basic trigonometric ratios. Give exact answers.

(a) $\sin x = \frac{3}{5}$

(b) $\cos x = \frac{2}{\sqrt{7}}$

(c) $\tan x = \frac{2}{\sqrt{5}}$

Q46. One of the angles in a right-angled triangle is 64°. The side opposite is 8.4 cm long. Find the lengths of the other sides.

Q47. Sketch the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for $0 \leq x \leq 360°$.

Q48. Find an angle $A°$, $180 < A < 360$ such that $\sin A° = \cos A°$.

Q49. Find an angle $A°$, $180 < A < 360$ such that $\cos B° = \cos 125°$.

Q50. Solve the following equations in the domain $0° \leq x \leq 360°$. Give your answers to two decimal places.

(a) $\cos x = -0.3$

(b) $\sin x = 0.4$

(c) $\tan x = 1.3$

Q51. *Sine rule & Cosine Rule*

(a) Find the missing angles in triangle ABC, where $BC = 6.82$, $AC = 7.38$ and $\hat{A} = 45°$.

(b) Find the missing side in triangle ABC, where $BC = 4.98$, $AC = 2.5$ and $\hat{C} = 131°$.

(c) The ship sailed from the port on a bearing of 115°. After 50 kilometres it changed the direction to a bearing of 062° from the point of change of direction. How far should it sail to be exactly east from its starting position? What will be its distance from the starting position?

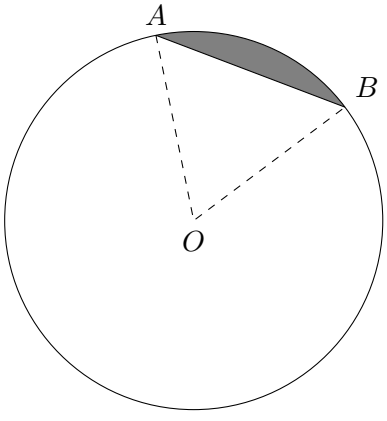
(d) Find the length of a diagonal of a regular pentagon with side of length equal 4.

Q52. *Sectors*

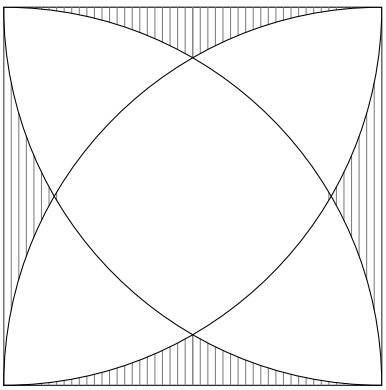
(a) Find the area of the shaded segment (see: diagram) if the circle's radius is r and the angle $A\hat{O}B = \alpha$ for:

(1) $r = 5$, $\alpha = 47°$ (2) $r = 6$, $\alpha = 52°$

Find the perimeter of the sector OAB using the values of r and α obtained above.



(b) Consider a square with side length 10 centimetres. Four arcs with radii 10 centimetres centred at vertices of the square are drawn inside a square as shown in the diagram. Find the area of the non-shaded region.



Year 12 sample questions for entry to A Level Further Mathematics

logarithms

Q53. Solve

(a) $\log_2 x = 4$

(b) $\log_x 8 = 3$

(c) $\log_5(x-1) + \log_5 2 = 2$

(d) $\log_3(2x-1) - \log_3 7 = 4$

(e) $\log_2 x + \log_2(x-6) = 4$

hidden quadratics

Q54. Solve

(a) $5^{2x} = 15 - 2(5^x)$

(b) $\log_2 x = \frac{2\log_2 x - 1}{\log_2 x}$

(c) $4\cos^2 x + 5\cos x - 1 = 0,\ 0 \leq x \leq 360°$

(d) $2\cos^2 x + 3\sin x - 3 = 0,\ 0 \leq x \leq 360°$ (Hint: $\sin^2 x + \cos^2 x = 1$)

(e) $4x^4 - 5x^2 + 1 = 0$

(f) $x^6 - 7x^3 - 8 = 0$

discriminant

Q55. For what values of k the equation $x^2 - 2(k+1)x - k + 1 = 0$ has two equal roots?

Q56. For what values of m the equation $x^2 + (m+1)x + m = 0$ has two different real roots?

Q57. For what values of m the equation $(m-3)x^2 + (3-m)x + 1 = 0$ has no real roots?

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polynomials

Q58. Sketch the following cubic functions.

(a) $x^2(x-1)$

(b) $(3x-1)(2x-1)^2$

(c) $(x+2)(x-1)(3x+1)$

(d) $(x-5)^3$

Q59. Solve the following inequalities.

(a) $x^3 - x^2 - 2x \leq 0$

(b) $-x^3 + x < 0$

(c) $-9x^3 + 3x^2 + 2x < 0$

modelling with trigonometry

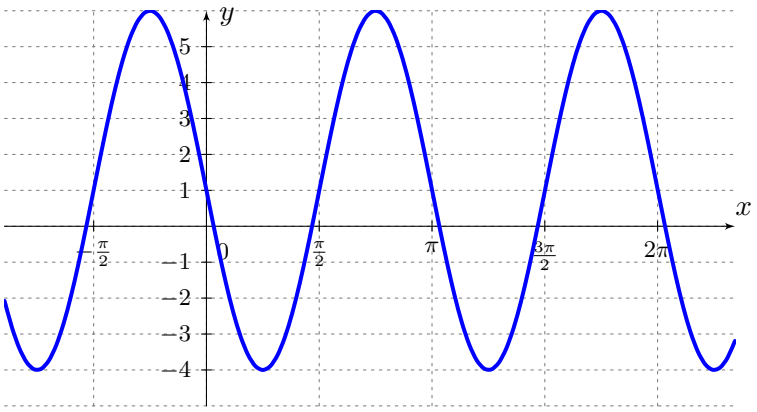
Q60. Sketch the graphs of.

(a) $y = 3\sin 2x$

(b) $y = -2\cos\left(\frac{1}{2}x\right)$

(c) $y = -\tan(2x - 90)$

Q61. The diagram below shows a graph of $y = A\sin Bx + C$. Find the values of A, B and C.



Q62. A metal ball is suspended on a string from the ceiling. It is pulled down and released, and then oscillates up and down. Its height, h centimetres above the floor, is modelled by the function

$$h = 210 + 4\sin((180t)^\circ),$$

where t is time in seconds after release.

(i) Find the height of the ball after 4 seconds.

(ii) What is the period of the motion?

(iii) Find the minimum height of the ball.

(iv) Find the first time at which the height is $208cm$.